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Abstract

We study two electricity markets connected by a fixed amount of cross-border capacity. The total amount of capacity is known to all electricity traders and allocated via an auction. The capacity allocated to each bidder in the auction remains private information. We assume that traders are faced with a demand function reflecting the relationship between electricity transmitted between the markets and the spot price difference. Therefore, traders act like Bayesian-Cournot oligopolists in exercising their transmission rights when presented with incomplete information about the competitors' capacities. Our analysis breaks down the welfare effect into three different components: Cournot behavior, capacity constraints, and incomplete information. We find that social welfare increases with the level of information with which traders are endowed.

Keywords: Cournot Oligopoly, incomplete information, capacity constraints, electricity markets, interconnector, cross-border trade

JEL classification: C72, D43, L13, L94

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1 Introduction

Efforts to liberalize European electricity markets led to unparalleled structural changes within the last 10 to 15 years. Directives and regulations issued by the European Commission aimed to open markets, ensure non-discriminatory third-party access to power grids¹ and enforce cross-border trading activities² in order to harmonize prices and to mitigate market power. Resulting from Article 6 of Regulation 1228/2003 –“*Network congestion problems shall be addressed with non-discriminatory market based solutions which give efficient economic signals [...]*”–, non-market-based congestion methods such as *first-come-first-serve* or *pro-rata* were replaced by market-based regimes like *implicit* and *explicit* auctions. In *explicit* auction regimes, the right to use cross-border capacity is sold first stage to market participants by a uniform-pricing auction. In a second stage, market participants then have to decide which share of their transmission rights to exercise in order to schedule a power flow from one market area to another.

Explicit auctions have been criticized mainly for two reasons. First, they might allow for exertion of market power. A firm might acquire capacity to block it or strategically misuse it to protect a dominant position in one regional market. Second, firms face incomplete information with respect to the demand for power transmission. Traders might just not know *ex ante* in which region excess demand (and therefore prices) are larger and might nominate capacity in the wrong direction. However, explicit auctions are still in place at many interconnectors.³

We add to the analysis of explicit auctions an additional source of inefficiency, namely the inefficiency arising from strategic usage of capacity under incomplete information with respect to the allocation of capacity among competing traders. To do so, we consider explicit auction regimes as two stage games: while transmission rights are sold to firms via an auction in the first step and auction results are made public, the actual utilization of transmission

¹European Union, Directive 54/EC (2003).

²European Union, Regulation EC No 1228 (2003).

³Examples are, among others, the interconnectors between France and the UK, France and Italy, Germany and Switzerland and Czech Republic and Poland.

capacity is determined by firms in the second step, in which firms essentially play a Bayesian-Cournot game. The strategic variable is a firm's utilization of transmission rights. We solely focus on the second stage of the game and argue why this is sufficient to demonstrate the inefficiency of the auction regime.

Since the total cross-border capacity is fixed, there is a strong stochastic dependency structure between the firms' transmission rights. Consequently, equilibrium strategies can not be derived analytically. Therefore, we solve the model numerically for the case of three firms, which is the simplest relevant model specification – in the case of two firms, the game is subject to complete information because total capacity is common knowledge.

It turns out that a unique equilibrium exists, provided that firms are symmetric. In particular, the equilibrium itself must be symmetric. This is achieved by showing that the best response function converges to a unique fixed point – as opposed to the standard form Cournot oligopoly, in which the best response function only converges as long as $n < 3$. This result enables us to implement a stable algorithm that converges to the unique symmetric Bayesian-Cournot equilibrium.

The simulation results show that in the unique Bayesian-Cournot equilibrium, firms fully exercise their transmission rights up to a certain threshold. When the transmission rights with which a firm is endowed exceed this threshold, a bend occurs, leaving afterwards the strategy increasing in a convex manner up to the firm's monopoly output.

Moreover, social welfare increases with the level of information. The increase in social welfare is driven by an increase in producer surplus – i.e., when firms have more information, they can coordinate better on total electricity transmission. In particular, firms have an incentive to commit on an industry-wide information sharing agreement *ex-ante*. Stabilizing total transmission reduces its variance, which in turn lowers consumer surplus. However, the effect on consumer surplus is small and can be ambiguous, depending on the model parameters.

The remainder of this paper is structured as follows. In Section 2, we provide a literature review. In Section 3, we explain cross-border economics, auction offices and further motivate the model. The model and analytical results

are presented in Section 4. The results of the numerical solution are presented and discussed in Section 5. Finally, Section 6 concludes.

2 Related Literature

The inefficiency of explicit auction regimes is unchallenged and has been documented in recent studies. Meeus (2011) describes the transition from explicit to implicit market coupling of the so-called *Kontek*-cable connecting Germany and the Danish island Zealand. He shows that implicit price coupling clearly outperforms explicit auctions. Gebhardt and Höffler (2013) find that cross-border capacity prices (first stage of the two-stage game) at the German-Danish and German-Dutch borders predict on average spot price differentials correctly, but with a lot of noise. Similar arguments are provided by Dieckmann (2008) and Zachmann (2008) who show that uncertainty about spot prices and timing of explicit auction regimes lead to a poor performance. For the German power market, Viehmann (2011) empirically shows the high volatility of spot prices also in comparison to their expected values.

While some of the literature mentioned above identifies market abuse as one possible reason for the inefficiencies observed, Bunn and Zachmann (2010) analytically derive cases in which dominant players, such as national incumbents, can maximize their profits by deliberately misusing cross-border capacities. The authors then analyze empirical data from the *IFA*-interconnector between France and UK and disclose flows against price differentials as well as unused capacity in the profitable direction in a significant number of hours. Additionally, Bunn and Zachmann (2010) provide a list of various design deficiencies contributing to the poor performance of explicit auction regimes. Finally, Turvey (2006) provides a broad overview about non-market and market-based congestion management methods and detailed information about South Eastern European markets.

The issue of incomplete information with respect to production capacities in Cournot oligopolies has recently been discussed by Richter (2013), who provides a characterization of equilibrium strategies when a firm's capacities are stochastically independent. Moreover, sufficient conditions for the existence

and uniqueness of a Bayesian-Cournot equilibrium are given. Bounded capacity is modeled by curtailing the firm's strategy space. We adopt this approach, since it ensures that the strategy spaces are compact and the expected payoff function is concave given a linear demand function, ensuring the existence of an equilibrium by Nash's theorem.

Regarding the issue of information sharing in oligopolies, literature focuses on Bayesian Cournot models in which there are no non-negativity constraints and no capacity constraints with respect to outputs. Provided the common prior belief is normally distributed, equilibrium strategies are linear (or affine) and closed-form solutions can be derived. An overview of these models is provided by Raith (1996). In all such models, firms face uncertainty with respect to marginal costs, or inverse demand, or both.

Most similar to the setting discussed in the paper at hand is the case of unknown costs, since costs as well as capacities are private values in which equilibrium strategies should be monotonous. Shapiro (1986) finds that in this case, firms have an incentive to share information, meaning that sharing information increases expected producer surplus. Moreover, he finds that consumer surplus decreases, whereas social welfare increases as a result of a positive net effect.

As outlined in the previous section, we obtain similar results as Shapiro, although the impact on consumer surplus is not that clear in the model developed. This is due to non-negativity and capacity constraints on outputs, leading to equilibrium strategies that are not affine. Thus, well-known results regarding information sharing can be reversed by introducing constraints – an issue that was addressed earlier by Maleug and Tsutsui (1998) and recently by Richter (2013).

3 Power Interconnectors

To further justify the use of the Cournot approach, we provide insights into interconnector economics and briefly introduce European auction offices and their information policies.

3.1 Interconnector Economics

While pools like the *PJM Market* in the US deal with regional supply and demand imbalances via nodal pricing, the predominant system in Europe can be described as a connection of market areas. In most cases, market areas that are connected by power interconnectors are equivalent to national borders.⁴

Today, the two prevailing mechanisms to allocate scarce cross-border capacities in Europe are *implicit* and *explicit* capacity auctions. With *implicit* auctions, also referred to as *market coupling* or *market splitting*, the auctioning of transmission capacity is implicitly integrated into the day-ahead exchange auctions of the connected market areas. Power exchanges can ensure welfare-maximizing cross-border flows between the market areas as they possess full information about all hourly supply and demand curves in the connected market areas and the available cross-border capacity.

When explicit capacity auctions are in place, the right to use cross-border capacity is sold in a first stage to market participants by a uniform-pricing auction, usually on a yearly, monthly and daily basis. In daily auctions, firms can bid for each hour of transmission capacity separately. In a second stage, market participants have to decide which share of their transmission rights to exercise in order to schedule a power flow from one market area to another.⁵

The basic interconnector economics are pictured in Figure 1, in which the relation between the *used transmission capacity* Q and the price spread P between two market areas is shown. When no transmission capacity is utilized ($Q = 0$), the price spread is at its maximum. The more capacity is booked to flow power from the low price area to the high price area, the smaller the price spread becomes. When the total available cross-border capacity \hat{t} is not sufficient to equalize prices (pictured left), total welfare is maximized at a price spread P^* and leads to $Q^* = \hat{t}$. However, provided the available cross-border capacity \hat{t} is more than sufficient to equalize prices (pictured right), the price spread P equals zero and $Q^* < \hat{t}$.

If *implicit* market coupling or market splitting is in place and no further

⁴Exceptions are Italy, the United Kingdom and the Scandinavian countries.

⁵A comprehensive overview of explicit and implicit cross border auctions is given by Kristiansen (2007) and Jullien et al. (2012).

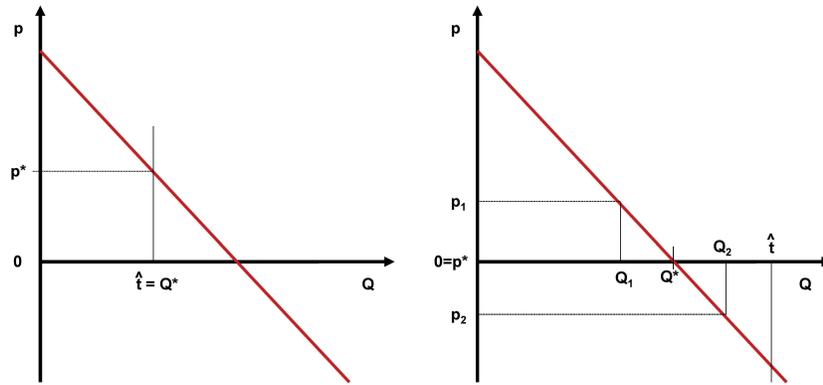


Figure 1: Basic economics of interconnectors

restrictions exist, the chosen quantity Q of cross border transmission flows is equal to Q^* for any given hour. The auction office knows the hourly aggregated supply and demand curves in both market areas and maximizes total welfare accordingly.

In the case of explicit auctioning, market participants who have acquired transmission rights determine the quantity Q . Empirical data shows that market participants do not choose the optimal quantity Q^* , especially when $Q^* < \hat{t}$ (Figure 1, right). As previously mentioned, there is a lot of noise in the empirical data due to the incomplete information about the demand for power transmission. However, when the assumption that firms play a Cournot game is valid, then firms must be undershooting on average, meaning that the outcome is *ex-ante* inefficient.

3.2 Auction Offices and Information Levels

Auction offices were recently subject to constant changes. Today, there are two main organizations in Europe, the *Capacity Allocating Service Company (CASC)*

and the *Central Allocation Office (CAO)*.⁶ Additionally, there are other platforms like *DAMAS*, *KAPAR* and the French TSO *RTE* that conduct daily cross-border auctions.⁷

In order to understand the inefficiencies in the second stage of explicit auction regimes, we first have a closer look at the auction offices and the information about the first-stage results passed to the traders. While some offices give detailed information about the number of successful bidders in the first stage (coincides with the number of firms in the second stage), others do not. The same holds true on how capacities are split among the firms. We analyze three explicit auction regime settings:

Complete information: The number of firms and their endowments with capacity are known to all firms,

Incomplete information: Each firm solely knows its own endowment, the number of competing firms is unknown,

Partial information: Each firm knows its own endowments and the number of other firms, but does not know their rival's endowment.

There is at least one auction office providing *complete information* for day-ahead capacity auction results. Using the *DAMAS System*, the Romanian TSO *Transelectrica*, for example, currently publishes the number of successful auction participants, their names and their allocated capacities.⁸ The *incomplete information* design, in which very little information about the number of success-

⁶CASC is currently operating daily cross-border capacity auctions at the Austrian-Swiss, Austrian-Italian, German-Swiss, French-Swiss, French-Italian, Greek-Italian and Swiss-Italian borders. Website: www.casc.eu. CAO is currently operating daily cross-border capacity auctions at the Austrian-Czech, Austrian-Hungarian, Austrian-Slovenian, Czech-German, Czech-Polish, German-Polish and the Polish-Slovakian borders. Website: www.central-ao.com. Last Update: 20th of September 2012.

⁷Daily cross-border auctions based on the *DAMAS* system are currently conducted at the French-English, Bulgarian-Romanian, and Hungarian-Romanian borders, among others. Daily cross-border auctions based on the *KAPAR* system operated by the Hungarian TSO *MAVIR* are currently conducted at the Hungarian-Croatian and the Hungarian Serbian borders. Last update: 20th of September 2012.

⁸*Transelectrica* is currently conducting explicit day-ahead auctions at the Bulgarian-Romanian and Hungarian-Romanian borders. <https://www.markets.transelectrica.ro/public>. Last update: 20th of September 2012.

ful bidders is published, is currently used by *RTE* at the French-Spanish Border and has been in operation at several other borders in the past. One prominent example was the German-French interconnector used before market-coupling started in November 2010. RTE merely publishes the number of successful bidders per day for daily auctions, meaning that firms know the maximum number of competitors for each hour but do not know how many competitors are endowed with a positive amount of capacity in a given hour. CASC and CAO currently publish *partial information*. They provide the number of successful bidders, but not precisely how capacities are split amongst them.

In the next section we present the general model framework, which is able to capture the information regimes as described above.

4 The Model

We consider a set of firms $N = \{1, 2, \dots, n\}$. Firms may face uncertainty with respect to the other firm's endowment of transmission capacity. In a Bayesian approach, a strategy of firm i is a decision rule that specifies a firm's amount of transmitted electricity for every possible information set with which the firm may be endowed. The amount of transmitted electricity corresponds to a firm's output in the Cournot model setting, and we use the terms *transmission* and *output* interchangeably.

We denote $T \subset \{0, 1, 2, \dots\}$ as the finite set of possible capacity levels and $\Omega = \prod_{n \in N} T$ as the set of possible states of nature. The common prior belief μ is a probability measure on Ω . An element of Ω , which is a capacity allocation among all n firms, is denoted by $\omega = (\omega_1, \omega_2, \dots, \omega_n)$. We assume that every firm is endowed with a production capacity exceeding zero with positive probability. The information with which a firm is endowed when making its output decision is described by a random variable T_i on Ω .⁹ A strategy is a function $q_i(T_i(\cdot))$ satisfying $q_i(T_i(\omega)) \leq \omega_i$. Lastly, we denote S_i as the strategy space of firm i and $S = \prod_{i=1}^n S_i$ as the space containing all strategy profiles.

As previously defined, $q_i(T_i(\omega))$ is the output of firm i . We let $Q(\omega) :=$

⁹The information sets of firm i are then the elements of the σ -algebra $\sigma(T_i)$ generated by T_i .

$\sum_{i=1}^n q_i(T_i(\omega))$ denote the overall output. The inverse demand function $P(Q)$ corresponds to the price difference between two electricity markets. We assume that P is linear and decreasing with total industry electricity transmission Q . We do not consider costs, since exercising transmission rights is costless.

The *state-dependent payoff function* u_i of firm i is given by

$$u_i(\omega, q_i, q_{-i}) = q_i(T_i(\omega))P(Q(\omega)). \quad (1)$$

A strategy profile $q \in S$ is a *Bayesian Cournot equilibrium* if for every i and $\tilde{q}_i \in S_i$ the *expected payoff function* is maximized,

$$E [u_i(\cdot, q_i, q_{-i})] \geq E [u_i(\cdot, \tilde{q}_i, q_{-i})], \quad (2)$$

meaning that in an equilibrium no firm has an incentive to unilaterally deviate from its strategy. Maximizing (2) is equivalent to maximizing the *conditional payoff expectation*, so that

$$E [u_i(\cdot, q_i, q_{-i}) | T_i(\omega)] \geq E [u_i(\cdot, \tilde{q}_i, q_{-i}) | T_i(\omega)] \quad (3)$$

for all $i \in N$ and all $\omega \in \Omega$.¹⁰

Remark 1. *Linearity of inverse demand ensures that the state-dependent payoff function (1) is concave in the output of firm i . Moreover, concavity is inherited by the expected payoff function (2) (Einy et al., 2010). Since a firm's strategy space is compact and convex, Nash's theorem implies the existence of an equilibrium.*

¹⁰See Harsanyi (1967-69) and Einy et al. (2002).

As previously mentioned, we analyze three schemes of information. In terms of the model formulation, the case of complete information corresponds to $T_i(\omega) = \omega$ for all $i \in N$ and all $\omega \in \Omega$. Thus, every firm is perfectly informed. When firms only know their own transmission capacity, then $T_i(\omega) = \omega_i$ holds. Finally, when information is partial, meaning that the number of active firms is known, then $T_i(\omega) = (\omega_i, F(\omega))$, where

$$F(\omega) = |\{i \in N : \omega_i > 0\}|.$$

In the next section we construct equilibrium strategies for the case of complete information. Moreover, for the case of three firms we provide a technique to numerically derive equilibrium strategies when information is incomplete.

4.1 Complete Information

This question of existence and uniqueness of equilibrium strategies in this setting is treated extensively in the literature.¹¹ However, we provide a constructive proof on existence and uniqueness, which coincidentally is helpful for the simulations. Speaking in terms of the model formulation, we discuss the case of $T_i(\omega) = \omega$ for all i and all ω .

We arbitrarily choose a capacity configuration $\omega = (\omega_1, \omega_2, \dots, \omega_n)$. Without loss of generality, we assume that $\omega_i \leq \omega_j$ if $i < j$. We let q_i denote the output of firm i and write $q = (q_1, q_2, \dots, q_n)$. The firm's equilibrium strategy of the corresponding unrestricted Cournot oligopoly is denoted by q^C . We define

$$q_1(\omega_1, \omega_2, \dots, \omega_n) = \min \{ \omega_1, q^C \}. \quad (4)$$

Firm 1 produces the n -firm Cournot quantity, whenever possible, and otherwise all of its capacity ω_1 . If $q(\omega_1, \omega_2, \dots, \omega_n) = q^C$, we define

$$q_j(\omega_j, \omega_1, \omega_2, \dots, \omega_{j-1}, \omega_{j+1}, \dots, \omega_n) = q^C$$

for all $j \geq 1$. If not so, we consider the $n - 1$ -firm oligopoly in which firms

¹¹See for example Bischi et al. (2010).

$i = 2, 3, \dots, n$ face residual demand resulting when firm 1 produces ω_1 . We let q_{n-1}^C denote the Cournot output of the corresponding unrestricted oligopoly and define

$$q_2(\omega_2, \omega_1, \omega_3, \dots, \omega_n) := \min \{ \omega_2, q_{n-1}^C \}.$$

By iteration, we obtain a strategy for every firm with the following property: There exists a threshold $k \in N$ so that $q_i(\omega_i, \omega_{-i}) = \omega_i$ for all $i < k$ and $q_i(\omega_i, \omega_{-i}) = q_k(\omega_k, \omega_{-k}) < \omega_k$ for all $i \geq k$, following from the construction procedure.

If in equilibrium there is a firm with a binding capacity restriction, the total output of the industry is lower compared to the output of the standard form Cournot oligopoly. This property is derived from the slope of the best response function r , which exceeds -1. If one firm decreases its output due to its capacity restriction, then the corresponding increase of the other firms is smaller. The following proposition sums up the well-known results we reconsidered in this section.

Proposition 1. *The strategy constructed above is the unique and symmetric complete information equilibrium of the Cournot oligopoly. If there exists an $i \in N$ such that $\mu(T_i < q^C) > 0$, then the expected total output in the complete information equilibrium is smaller compared to the total output of the unrestricted Cournot oligopoly.*

All proofs are provided in the Appendix of the chapter.

4.2 Incomplete and Partial Information

The results provided in this section cover both the case of incomplete information and the case of partial information defined on page 8. Since we seek to solve the model numerically, we provide an algorithm converging to a unique equilibrium solution, which then must be symmetric.

While equilibrium strategies can be explicitly constructed in the case of complete information, as demonstrated in the last section, this task is challenging when information is incomplete. In the very general model setting presented on Page 9, equilibrium strategies can be of any shape since the common

prior belief is left unspecified.¹²

However, in the context of exercising cross-border capacity, we can impose two restrictions on the common prior belief. First, firms are *ex-ante* symmetric by assumption. This leads to the following requirement:

$$\mu(T_i = t) = \mu(T_j = t) \text{ for all } t \in T \text{ and } i \neq j. \quad (5)$$

Second, we explicitly allow for firms to be endowed with zero capacity with positive probability. In particular, given that firm 1 is endowed with some capacity level t , then, with positive probability, firm 2 is endowed with zero capacity as long as there are at least three firms participating. This leads to

$$\text{If } n > 2, \text{ then } \mu(T_2 = 0 | T_1 = t) > 0 \text{ for all } t \in T. \quad (6)$$

Conditions (5) and (6) do not sufficiently specify the common prior belief to allow for an analysis of the shape of equilibrium strategies. To provide intuition for that, we consider the following construction procedure for the common prior belief. Let $\tilde{\mu}$ be an arbitrarily chosen probability measure on the product space $\prod_{i=1}^n T$ such that $\tilde{\mu}$ meets conditions (5) and (6). If T_i denotes the capacity with which firm i is endowed and if \hat{t} denotes the overall cross-border capacity, we can define

$$\mu(\cdot) := \tilde{\mu}(\cdot \mid \sum_{i=1}^n T_i = \hat{t}).$$

Thus, we can choose almost any distribution for $\tilde{\mu}$ and obtain the corresponding common prior belief μ . Even for a simple $\tilde{\mu}$, the conditional distribution μ is difficult to handle.

However, conditions (5) and (6) enable us to prove the existence of a unique Bayesian-Cournot equilibrium for the case of three firms. We show that under conditions (5) and (6), the industry's best response function \tilde{r} is a contraction mapping, meaning that if we iterate the best response function, then the sequence we obtain converges to the unique equilibrium solution.¹³

Therefore, we derive the best response function of the model. For a given

¹²See Richter (2013) for an example.

¹³More precisely, there exists $\theta < 1$ and a metric d on the space S of strategy profiles so that

$$d(\tilde{r}(q), \tilde{r}(q')) \leq \theta d(q, q')$$

strategy profile $q = (q_1, q_2, \dots, q_n)$, we write $q_{-i} = \sum_{j \neq i} q_j$ and define for $t \in T$ and $i \in N$

$$\tilde{r}_i(t, q_{-i}) = \min \{t, r(E[(q_{-i}|T_i = t])]\}.$$

Thus, $\tilde{r}_i(t, q)$ is the best response function of firm i when it is endowed with capacity t , given that the other firms apply q_{-i} . This stems from linear demand, since then the best reply function r of the unrestricted Cournot oligopoly only depends on the expected output of the other firms $j \neq i$. We define

$$\tilde{r}(q) := (\tilde{r}_i(t, q_{-i}))_{i \in N, t \in T}$$

to be the vector of best responses in each state and for each firm. Then a fixed point of \tilde{r} is an equilibrium. Theorem 1 states that the iterated best response function converges to the unique fixed point. While we cannot derive equilibrium strategies analytically, Theorem 1 implies that we can numerically implement the iterated best response algorithm for any common prior belief and obtain the unique equilibrium solution.

Theorem 1. *Under conditions (5) and (6) and when $n \leq 3$, for any q_0 the sequence*

$$q(n) := \tilde{r}(q(n-1))$$

converges to the unique fixed point q that does not depend on the choice of q_0 . In particular, a unique equilibrium exists, which then must be symmetric.

5 Numerical Solution to the Model

We solve the model numerically and compare the corresponding market outcomes by means of social welfare, producer surplus and consumer surplus for the three information regimes *incomplete information (II)*, *partial information (PI)* and *complete information (CI)*. For the simulation, we assume that inverse demand is given by $p(q) = 6 - q$. We allow for 21 capacity levels, starting at 0 and ending at 5. The distance between any two capacity levels is constant and

for all strategy profiles q, q' . Moreover, S needs to be complete with respect to d . Then, the sequence $x_n := \tilde{r}(x_{n-1})$ converges to some element x that does not depend on x_0 . Completeness with respect to d ensures that x is an element of S .

equal to 0.25. Lastly, we assume that μ is uniformly distributed on the set of feasible capacity levels.

5.1 Equilibrium Strategies

In Figure 2A, the equilibrium strategy for the incomplete information setting is pictured. On the horizontal axis, the capacity with which a firm is endowed is plotted and on the vertical axis, we can see the corresponding output. The symmetric equilibrium strategy is strictly increasing with a firm's capacity. As in the i.i.d.-case analyzed by Richter (2013), firms fully utilize their capacity up to a threshold. Then, a bend occurs and the strategy is increasing up to the monopoly output in a convex manner. Indeed, a firm must produce its monopoly output when it is endowed with maximum capacity, since then the firm is facing a monopoly with complete information.

Next, we consider Picture B, in which the *PI*-equilibrium strategy q^{PI} is plotted (to some extent). Because q^{PI} is a function of two arguments (capacity of a firm and number of active players), we cannot directly plot it in Figure 2, and a three-dimensional chart is unfortunately not instructive. Therefore, we define q_{\min}^{PI} to be

$$q_{\min}^{PI}(\omega_i) = \min\{q^{PI}(T_i(\tilde{\omega})) \mid \tilde{\omega}_i = \omega_i\}.$$

Thus, for a given capacity level ω_i , we pick the smallest equilibrium output among all possible numbers of active players given ω_i . The number q_{\max}^{PI} is defined accordingly and, as seen in the example, q_{\max}^{PI} equals q^{PI} if and only if there are two or less active firms. In the example, a firm has complete information when knowing that there is only one competitor.

We can see that the *PI*-strategy exceeds the *II*-strategy on a certain range (if the number of active firms is low) and the other way around (if the number of active firms is small). The range $[2, 3]$ corresponds to the event $(2, 3, 0)$ (or a permutation) in which two firms produce their two-player Cournot quantity. Moreover, for large capacity values, both strategies converge: If firm 1 is endowed with a sufficiently large amount of capacity, the other firms fully utilize their capacity in both information settings.

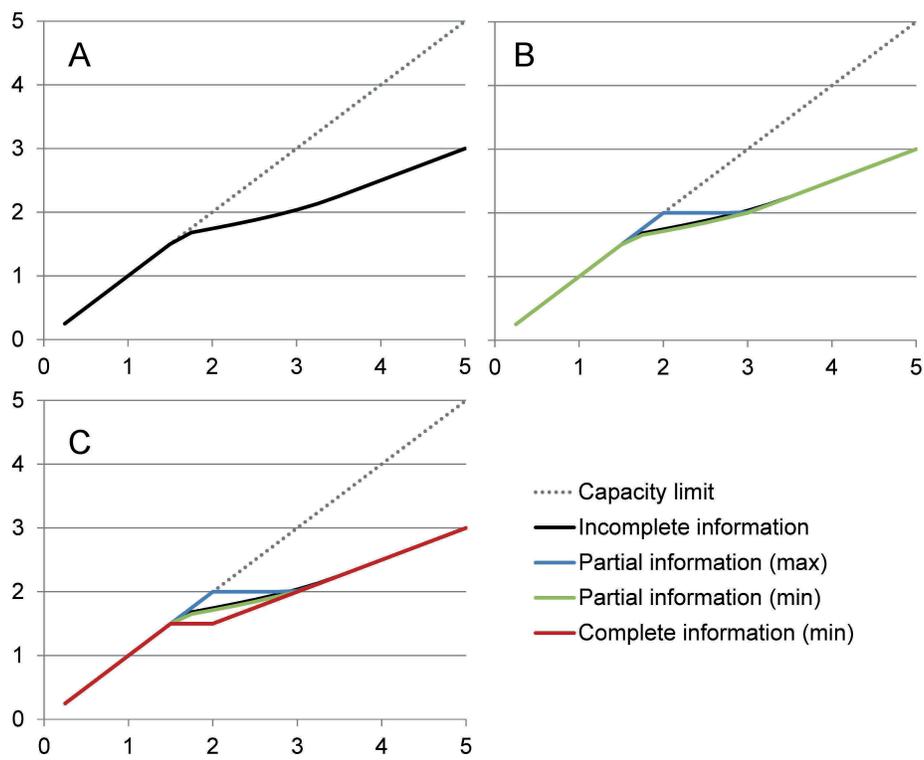


Figure 2: Numerically derived equilibrium strategies

Lastly, we depict a similar modified strategy for the case of complete information in Picture C. The corresponding maximal strategy q_{\max}^{CI} coincides with q_{\max}^{PI} because in both cases, firms face complete information. The corresponding minimum strategy q_{\min}^{CI} is smaller than the other strategies, since under complete information, a firm can protect itself against the case in which all three firms have roughly the same amount of capacity. In fact, in the range $[1.5, 2]$, the strategy q_{\min}^{CI} corresponds to the case in which every firm produces its Cournot quantity, which corresponds to the event $(2, 1.5, 1.5)$ (or a permutation).

5.2 Social Welfare

In this section, we analyze expected social welfare for the different information regimes and different demand intercepts. We express the expected welfare achieved under a given scheme of information and for a given demand intercept as a share of the maximal achievable welfare. When the demand intercept exceeds total capacity, welfare is maximized if and only if every firm utilizes all of its capacity. When the demand intercept is smaller than total capacity, welfare is maximized at the demand intercept.

As previously defined, the random variable $Q(\omega)$ denotes the industry's realized output. Consumer surplus is equal to $CS(\omega) := Q(\omega)^2/2$ and producer surplus is given by the aggregate industry profit $PS(\omega) := Q(\omega)P(Q(\omega))$. We define realized social welfare to be $CS(\omega) + PS(\omega)$.

Figure 3 shows the expected welfare for the different schemes of information. On the horizontal axis, the demand intercept is plotted. On the vertical axis, we can see the expected share of maximum achievable welfare (Figure 3 B is an enlargement of Figure 3 A).

The expected welfare in the complete information regime and the partial information scheme coincide when the demand intercept is sufficiently small. In this setting, firms do not fully utilize their capacities (as long as capacity is exceeding zero). Therefore, firms have complete information when they are informed about the number of active firms.

Furthermore, relative expected welfare approaches unity as the demand

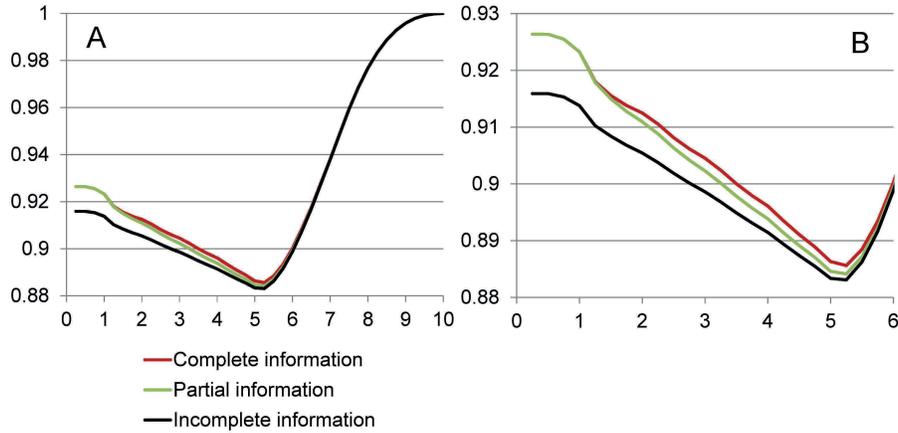


Figure 3: Effects of information sharing on social welfare

intercept approaches 10 in all information regimes. Apparently, this is because then every firm fully utilizes its capacity in every information regime and in every state of nature. In this case, we have defined the maximum achievable welfare to be full utilization of total capacity. Via similar reasoning, the curve is increasing on the right-hand side of its local minimum. Therefore, relative expected social welfare is high when either capacity limits are rarely active (when the demand intercept is small, case 1) or when they are rarely redundant (when the demand intercept is high, case 2).

Equivalently speaking, expected social welfare is low if, with high probability, a firm with a large capacity can act as a monopolist on residual demand, since the other firms have little capacity and thus fully utilize it. In this case, the dominant firm leaves a large share of capacity unused. This follows from the slope of the best response function, which is equal to $-1/2$.

The impact of the slope of the best response function on total electricity transmission becomes smaller in case 1 and vanishes in case 2 as defined above. In case 1, in which the demand intercept is relatively small compared to total cross-border transmission capacity, firms do not fully utilize their capacity, since their capacity limits exceed the Cournot quantity of the unrestricted game. Therefore, if the demand intercept is sufficiently small, partial information is equivalent to complete information, whereas firms face uncertainty with

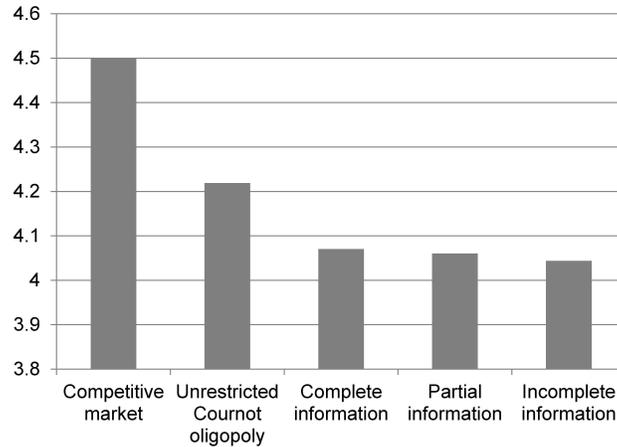


Figure 4: Expected welfare in different information regimes

respect to the number of active firms in the case of incomplete information.

In case 2, in which the demand intercept is relatively large compared to total cross-border transmission capacity, every firm fully utilizes its capacity, regardless of the observed capacity allocation. In this case, the equilibria of all three information regimes coincide.

Lastly, Figure 3 shows that social welfare increases with the level of information. This is the main result of the paper. Figure 4 compares expected welfare for different settings for the case in which the demand intercept equals 3. In the competitive market outcome, total output equals the demand intercept. Consumer surplus and social welfare coincide, since marginal costs are zero, and are equal to $3^2/2 = 4.5$. The outcome of the unrestricted Cournot oligopoly leads to an output of $9/4$. This leads to a dead weight loss of $(3 - 9/4)^2/2 = 9/32$, thus implying that social welfare is equal to $4.5 - 9/32 \approx 4.22$. To sum up, we can identify three driving forces reducing welfare.

First, Cournot behavior of firms reduces welfare, a well-known fact that holds in any Cournot oligopoly setting.

Second, capacity constraints reduce welfare, even when total capacity exceeds the demand intercept and firms have complete information. This result is

already indicated by Proposition 1, which states that in the presence of capacity constraints, the expected total transmission of electricity declines. The effect on welfare is initially unclear; however, Figure 4 shows that due to capacity constraints, welfare decreases.

Third, a reduction in information reduces welfare. The information effect is systematic but small; however, if we chose a common prior belief with a higher variance, the effect would probably become stronger.¹⁴ The next two sections seek to explain the information effect on social welfare. The main driving force is the variance of total electricity transmission.

5.3 Consumer and Producer Surplus

We demonstrate that the increase of social welfare induced by information sharing is driven by an increase in producer surplus, whereas the effects on consumer surplus are small and partly ambiguous. When firms are better informed, they coordinate better on total industry output. This lowers the variance of total output, which decreases consumer surplus. This effect on consumer surplus is clearly observable when comparing the incomplete information equilibrium with the complete information equilibrium. However, the effect is less clear when we compare the partial information equilibrium with the complete information equilibrium.

5.3.1 Producer Surplus

As before, we calculate a relative number: We define the maximum achievable producer surplus to be the minimum of the maximal capacity and the aggregate industry output of the standard form Cournot oligopoly. Then, we consider the ratio of expected producer surplus and maximum achievable producer surplus.

Figure 5 shows that the effect on producer surplus is similar to the effect on social welfare – producer surplus increases with the level information. However, there are states of nature in which producer surplus can decrease due to information sharing: When there are two firms A and B that do not fully utilize their capacity in the incomplete information equilibrium, and when some firm

¹⁴Richter (2013) discusses the impact of the variance of the common prior believe on results of information sharing in a similar context.

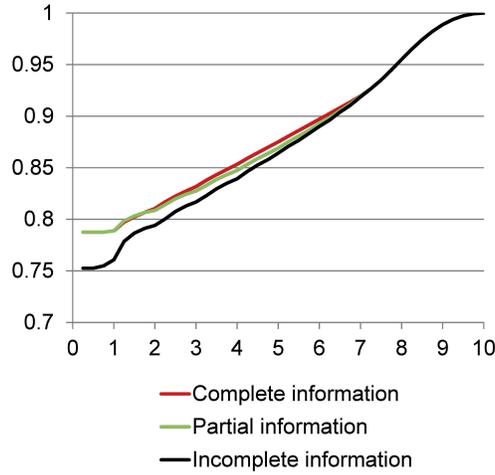


Figure 5: Effects of information sharing on producer surplus

C is endowed with zero capacity, then revealing this information induces firms A and B to increase their output. This is because the incomplete information output of firms A and B takes into account the possibility that there are three active firms rather than two. To give an example based on the simulation results, we consider the case in which the demand intercept is equal to 1. Firm A has a capacity that is equal to 2 and firm B has a capacity that is equal to 5. Under incomplete information, firm A produces 0.253, whereas firm B produces 0.296. That is to say, A and B take into account that the remaining capacity is (evenly) split up between two firms, which is why A and B produce less than the Cournot quantity, which is equal to 0.333. These equilibrium outputs lead *ex-post* to payoffs that are equal to 0.114 and 0.133, respectively. The complete information output of A and B equals 0.333, leading to a payoff that is equal to 0.112.

Similarly, there are states of nature in which producer surplus increases when information is shared. The simulation results show that this is always true as long as there are one dominant firm and two firms with little or zero capacity. Then, the small firms overestimate total industry output under incomplete information, and, as a consequence, their outputs are *ex-post* too low. Therefore, when information is shared, small firms increase their output. Be-

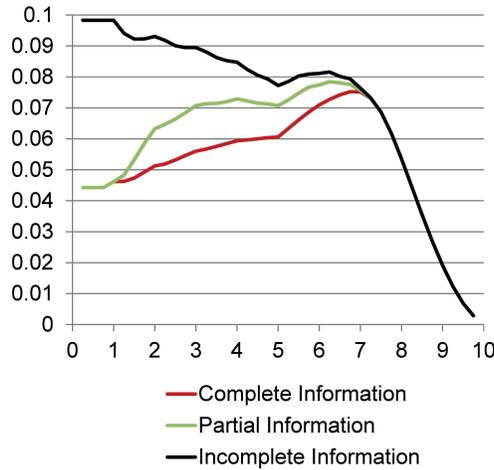


Figure 6: Effects of information sharing on the standard deviation of total industry output

cause total industry output is relatively low due to the presence of a large firm, the marginal revenue of an increase of output is positive. Thus, the small firms gain from sharing.

Notice that in every information regime the outputs of the firms are negatively correlated. This is because if a firm is endowed with a large share of cross-border capacity, the other firms are endowed with little capacity. As a consequence, the variance of total output decreases.

Apparently, the absolute value of the correlation of outputs increases with the level of information, regardless of the choice of the common prior belief. This is because firms transmit some “average” amount of electricity when they have little information. Figure 6 shows that the variance of total output is decreasing with the level of information. Since consumer surplus is increasing with the variance of total industry output (see Richter (2013) or Shapiro (1986)), Figure 6 indicates that consumer surplus decreases with the information with which firms are endowed.

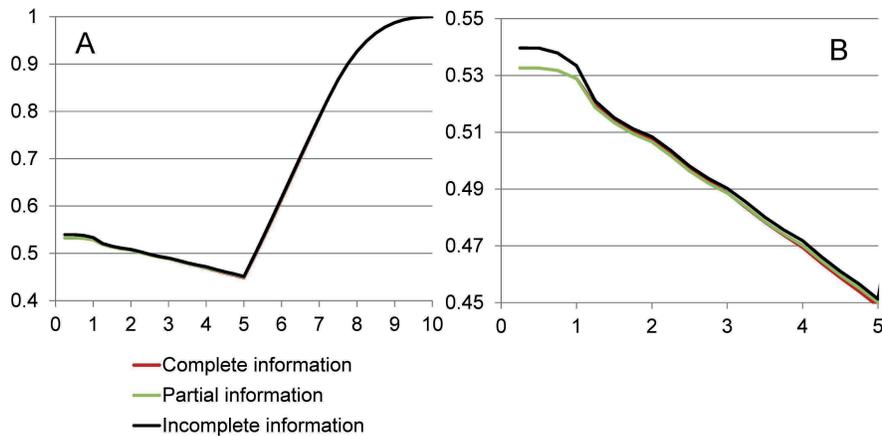


Figure 7: Effects of information sharing on consumer surplus

5.3.2 Consumer Surplus

Figure 7 A shows that consumer surplus varies with the demand intercept in a similar fashion as social welfare. Starting at 0.53, a local minimum of 0.45 is attained when the demand intercept equals 5. Apparently, the effect of different information regimes on consumer surplus is small.

Figure 7 B enlarges the range $[0, 5]$. The expected consumer surplus in the incomplete information setting weakly exceeds both the complete information and partial information consumer surplus. However, in the case of partial information, consumer surplus can be above and below consumer surplus resulting from complete information. Thus, a clear statement regarding the impact of information sharing on consumer surplus can not be obtained.¹⁵ However, Figure 6 shows that we can identify one stable result with respect to consumer surplus: The standard deviation of total output is decreasing with the level of information, which in turn decreases consumer surplus. To sum up, the impact on consumer surplus is small, and increasing information tends to reduce consumer surplus. The same holds true for expected electricity transmission. This follows from the fact that the variance of total output is decreasing and from the fact that consumer surplus is increasing with both variance of total output

¹⁵This is a common issue, seen for example in Raith (1996).

and expected total output.

6 Results and Discussion

We analyzed the strategic behavior of firms endowed with transmission rights that arises when transmission capacity between electricity markets is explicitly auctioned. In doing so, we perceived the strategic behavior of firms as a Cournot oligopoly in which firms face incomplete information with respect to the other firms' transmission rights.

Thereby, total cross-border capacity is common knowledge, which enables a firm to calculate the conditional distribution of the other firms' transmission rights given its own amount of transmission rights (the case of *incomplete information*). Moreover, we allow for an information regime in which the number of firms endowed with a positive amount of transmission rights is also revealed to the firms (the case of *partial information*).

For the case of three or less firms, we have shown that the best response function is a contraction, a result that is specific to the special setting under consideration. The best response function converges to the unique Bayesian Nash equilibrium, which, in particular, must then be symmetric. Because the best response function converges, we were able to calculate equilibrium solutions by means of simulation and to perform a sensitivity analysis with respect to the demand intercept. Moreover, we calculated the equilibrium for the case of complete information.

By comparing the equilibria for the three information regimes, we find that revealing information to firms increases social welfare. The increase of social welfare is driven by an increase in producer surplus. The states of nature that potentially diminish producer surplus are overcompensated by states of nature in which producer surplus increases. Since information sharing increases the negative correlation of the firms' outputs, the variance of total industry output decreases.

Although a decrease of the variance of total industry output in general decreases consumer surplus, the effect on consumer surplus is smaller than on producer surplus. We find that expected consumer surplus decreases when

moving from the incomplete information equilibrium to the partial information or to the complete information equilibrium. However, when moving from the partial information equilibrium to the complete information equilibrium, the effect on consumer surplus is ambiguous. As a consequence, the same holds for total electricity transmission.

Thus, we identified three forces regarding capacity auctions that diminish social welfare: First, firms play a Cournot game, which prevents an efficient market outcome. Second, the presence of capacity constraints further reduces social welfare. This is derived from the slope of a firm's best response function, which exceeds -1 : When a firm with little capacity fully exercises its transmission rights, its lack of transmission is not fully compensated by those firms endowed with a large amount of transmission rights. Third, incomplete information reduces welfare as well, as in the presence of incomplete information, firms exercise their transmission rights less aggressively.

As mentioned in the introduction, explicit capacity auctions are in fact a two-stage game. In the first stage, the transmission rights are auctioned. Then, firms are informed about their own amount of transmission rights (and, depending on the auction office, the number of active firms). In the second stage, firms exercise their transmission rights. The model analyzed in the paper at hand could be expanded to a two-stage game such as the following example.

Before the first step of the auction process is conducted, firms observe signals about a common value, for example the demand intercept of the inverse demand function. The action space of the first stage can be modeled via linear bidding functions that are decreasing, mapping transmission capacity to a price. The horizontal intercept of each firm's bidding function could be modeled as an increasing function of the firm's signal. The market operator then selects the highest bids and assigns transmission rights to the firms. When firms make their output decisions in the second step, the transmission rights of the other firms are stochastic – the corresponding distribution is induced by the distribution of the signals observed by the firms before the first step of the auction was conducted. Thus, the second stage game is equivalent to the game analyzed in the paper at hand. The results on the three driving forces diminishing social welfare should be stable even when the problem is modeled as a

two-stage game.

As previously mentioned, implicit auction regimes clearly outperform explicit auction regimes. Nevertheless, as long as explicit auction regimes are still in place, we recommend that auction offices provide as much information as possible about the first stage results in order to maximize social welfare.

Appendix

Proof of Proposition 1

To show that q is an equilibrium, we choose the smallest number $k \in N$ so that $q_k(\omega_k, \omega_{-k}) < \omega_k$. Then $q_k(\omega_k, \omega_{-k})$ is firm k 's best response by definition. Since a firm $i > k$ minimizes the same payoff function as firm k does, $q_i(\omega_i, \omega_{-i}) = q_k(\omega_k, \omega_{-k})$ is the best response of firm i as well. Any firm $i < k$ can not increase its output and does not have an incentive to decrease its output because $q_i(\omega_i, \omega_{-i}) < q_k(\omega_k, \omega_{-k})$. Furthermore, firm k does not have an incentive to decrease its output.

To show that the equilibrium is unique, we consider $\tilde{q} \neq q$ to be another equilibrium and denote i as the smallest number such that

$$\tilde{q}_i(\omega_i, \omega_{-i}) \neq q_i(\omega_i, \omega_{-i}).$$

Without loss of generality, we assume that $i = 1$. First, we consider the case in which

$$\tilde{q}_1(\omega_1, \omega_{-1}) < q_1(\omega_1, \omega_{-1}).$$

This implies

$$\tilde{q}_1(\omega_1, \omega_{-1}) < \omega_1,$$

which in turn leads to

$$\tilde{q}_j(\omega_j, \omega_{-j}) = \tilde{q}_j(\omega_i, \omega_{-i})$$

for all $j > i$. But then

$$q^C = \tilde{q}_1(\omega_1, \omega_{-1}) < q_1(\omega_1, \omega_{-1}),$$

contradicting (4).

Second, when

$$\tilde{q}_1(\omega_1, \omega_{-1}) > q_1(\omega_1, \omega_{-1}),$$

we conclude

$$q_1(\omega_1, \omega_{-1}) < \omega_1$$

and thus

$$q_j(\omega_j, \omega_{-j}) = q_i(\omega_i, \omega_{-i})$$

for all $j > i$, meaning that q is the standard form of the Cournot oligopoly equilibrium, which is unique, thus implying that \tilde{q} can not be an equilibrium.

To show that the statement holds in the case of duopoly, we let r denote the best response function of the unrestricted Cournot duopoly. We choose $\omega \in \Omega$ arbitrarily and assume that firm 1 produces ω_1 and firm 2 produces $r(\omega_1) < \omega_2$ in the unique equilibrium. Then, since $r(q^C) = q^C$,

$$\omega_1 + r(\omega_1) \leq 2r(q^C)$$

if and only if

$$r(\omega_1) \leq 2q^C - \omega_1,$$

which is equivalent to

$$r(\omega_1) - r(q^C) \leq q^C - \omega_1. \quad (7)$$

The decrease of production by firm 1 must overcompensate the increase of production by firm 2, which is true: Equation (7) holds because $r' > -1$. Without loss of generality, we assume that $\mu(T_1 < q^C) > 0$, which yields the given statement.

The result easily translates to the case of an oligopoly. We arbitrarily choose a capacity configuration $(\omega_1, \omega_2, \dots, \omega_n)$. Again, we assume that $\omega_i \leq \omega_j$ if $i \leq j$. Choose k so that $q(\omega_{k-1}, \omega_{-k-1}) = \omega_{k-1}$ and $q(\omega_k, \omega_{-k}) < \omega_k$. Define the capacity configuration $(\tilde{\omega}_1, \tilde{\omega}_2, \dots, \tilde{\omega}_n)$ by $\tilde{\omega}_i := \omega_i$ if $i < k - 1$ and for $i \geq k - 1$ choose $\tilde{\omega}_i$ large enough so that in the corresponding equilibrium $q(\omega_{k-1}, \omega_{-k-1}) = \omega_{k-1} < \tilde{\omega}_{k-1}$, meaning that when moving from $(\omega_1, \omega_2, \dots, \omega_n)$ to $(\tilde{\omega}_1, \tilde{\omega}_2, \dots, \tilde{\omega}_n)$ the former active capacity restriction of firm $k - 1$ becomes inactive, whereas all other active capacity restrictions remain as they are. Having established this, it is sufficient to show that the

total output of the industry with respect to the former capacity configuration is smaller than the output of the industry with respect to the new capacity configuration $(\tilde{\omega}_1, \tilde{\omega}_2, \dots, \tilde{\omega}_n)$. But this follows from the case of duopoly: We can either focus on the residual game in which we neglect firms $1, 2, \dots, k-2$ or we assume without loss of generality that $k=2$.

Proof of Theorem 1

Any feasible strategy profile q is an element of

$$S = \prod_{j \in N} \{q : T \rightarrow \mathbb{R}_+ | q(t) \leq t \text{ for all } t \in T\}.$$

If we define

$$d(q, q') = \max_{j \in N, t \in T} |q_j(t) - q'_j(t)|,$$

then (S, d) is a complete metric space. Thus, it is sufficient to show that \tilde{r} is a contraction with respect to d , since then Banach's fixed-point theorem establishes that \tilde{r} has a unique fixed point. Therefore, it remains to be shown that there exists $0 \leq \theta < 1$ so that

$$d(\tilde{r}(q), \tilde{r}(q')) \leq \theta d(q, q')$$

for every $q' \in S$ such that $q' \neq q$.

We define

$$p := \min_{t \in T} \{\mu(T_2 = 0 | T_1 = t)\}$$

and

$$\theta := \frac{(1-p)(j-1)}{2}.$$

Clearly, if $j=2$, then $\theta < 1$. If $j=3$, then $p > 0$ due to (6) and thus $\theta < 1$ as well. We choose $s \in T$ and $i \in N$ such that

$$\begin{aligned} d(\tilde{r}(q), \tilde{r}(q')) &= |\tilde{r}_i(s, q_{-i}) - \tilde{r}_i(s, q'_{-i})| \\ &= \left| \min\{s, r(E[q_{-i} | T_i = s])\} - \min\{s, r(E[q'_{-i} | T_i = s])\} \right|. \end{aligned} \tag{8}$$

If (8) = 0, then $q = q'$, which contradicts the assumption that $q \neq q'$. Thus, we must have (8) > 0. In particular, either

$$r(E[q_{-i}|T_i = s]) < s$$

or

$$r(E[q'_{-i}|T_i = s]) < s$$

or both. For the last case when both capacity limits are not active, we obtain

$$(8) = \frac{1}{2} \left| E[q_{-i} - q'_{-i}|T_i = s] \right| \leq \frac{(1-p)(j-1)}{2} d(q, q') = \theta d(q, q'),$$

since q_{-i} and q'_{-i} differ at most with probability $1 - p$, and the difference can never exceed $(j - 1)d(q, q')$ by definition. If only one capacity constraint is active, say $r(E[q_{-i}|T_i = s]) = s$ without loss of generality, we get

$$\begin{aligned} (8) &= s - r(E[q'_{-i}|T_i = s]) \\ &\leq r(E[q_{-i}|T_i = s]) - r(E[q'_{-i}|T_i = s]) \end{aligned}$$

and the proposed statement follows from the case case where both capacity limits are not active.

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